“Advanced Automatic Control MDP 444”

If you have a smart project, you can say "I'm an engineer"

Lecture 7

Staff boarder

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Advanced Automatic Control
MDP 444

- **Lecture aims:**
  - Be familiar with the design of lead and lag compensators using Bode plot methods
  - Understand the design of a control system is concerned with the arrangement, or the plan, of the system structure and the selection of suitable components and parameters

Bode Plots

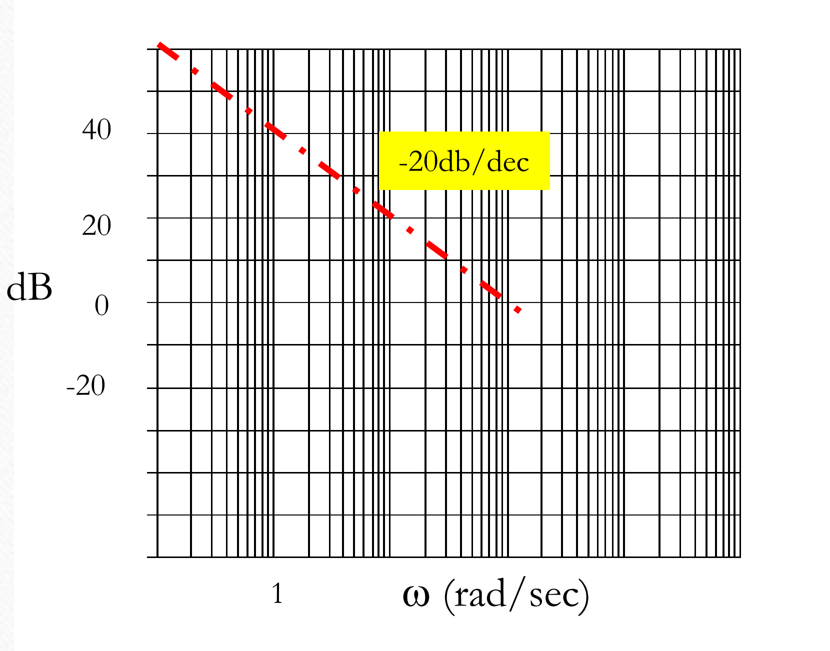
Example : Given the transfer function. Plot the Bode magnitude.

$$G(s) = \frac{100(1+s/10)}{s(1+s/100)^2}$$

Consider first only the two terms of $\frac{100}{jw}$

Which, when expressed in dB, are; 20log100 – 20 logw.
This is plotted below.

The .... is a tentative line we use until we encounter the first pole(s) or zero(s) not at the origin.



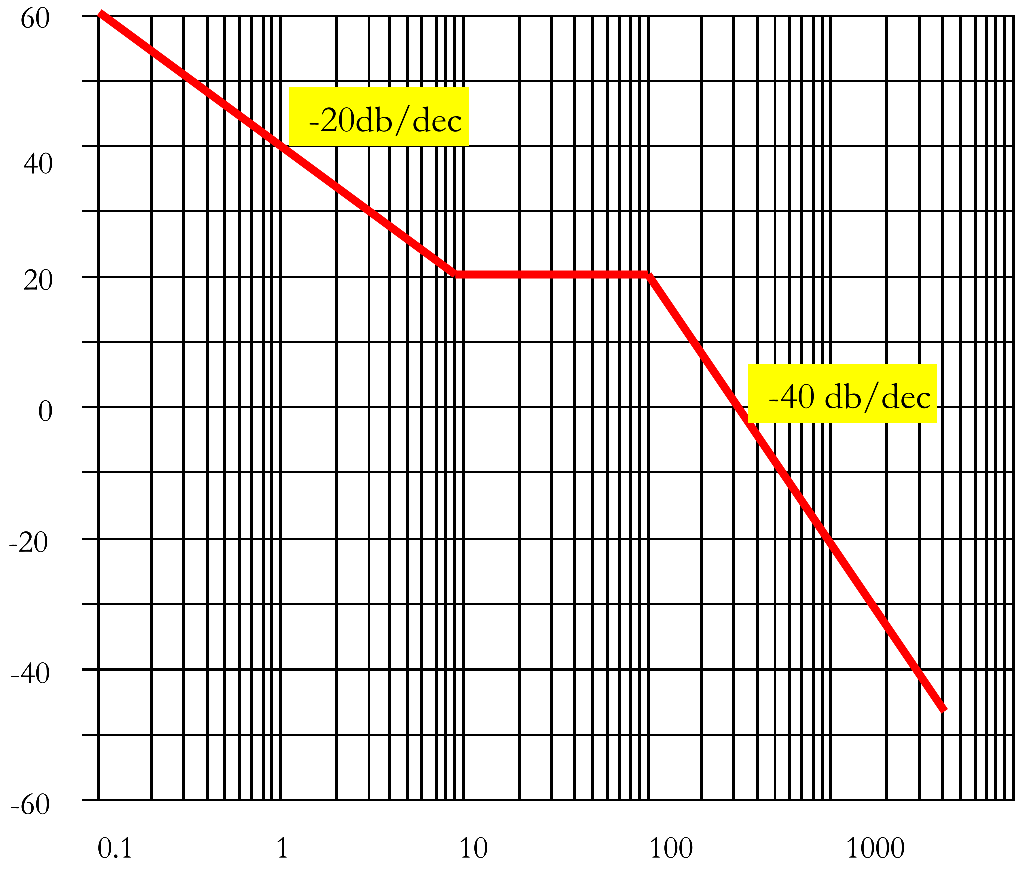
Bode Plots

Example :

The completed plot is shown below.

$$G(s)=\frac{100(1+s/10)}{s(1+s/100)^2}$$

dB Mag



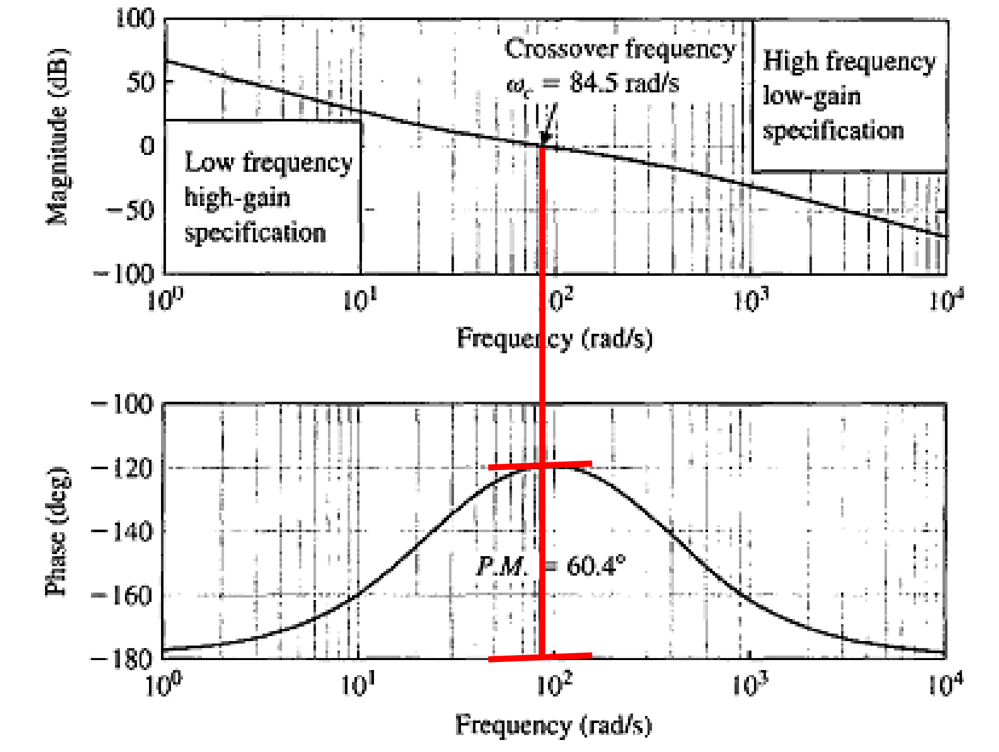
Phase (deg)

Phase

$$\angle G(jw)=\tan^{-1}(w/10)-\tan^{-1}(\infty)-\tan^{-1}(w/100)$$

Stability in Frequency Response

- G.M = +Ve if belwo zero of magnitude
- P.M = +Ve if above -180 of phase
- Stable if G.M & P.M are +ve
- Unstable if G.M & P.M are –ve
- Critical stability if G.M & P.M are in opposite signs



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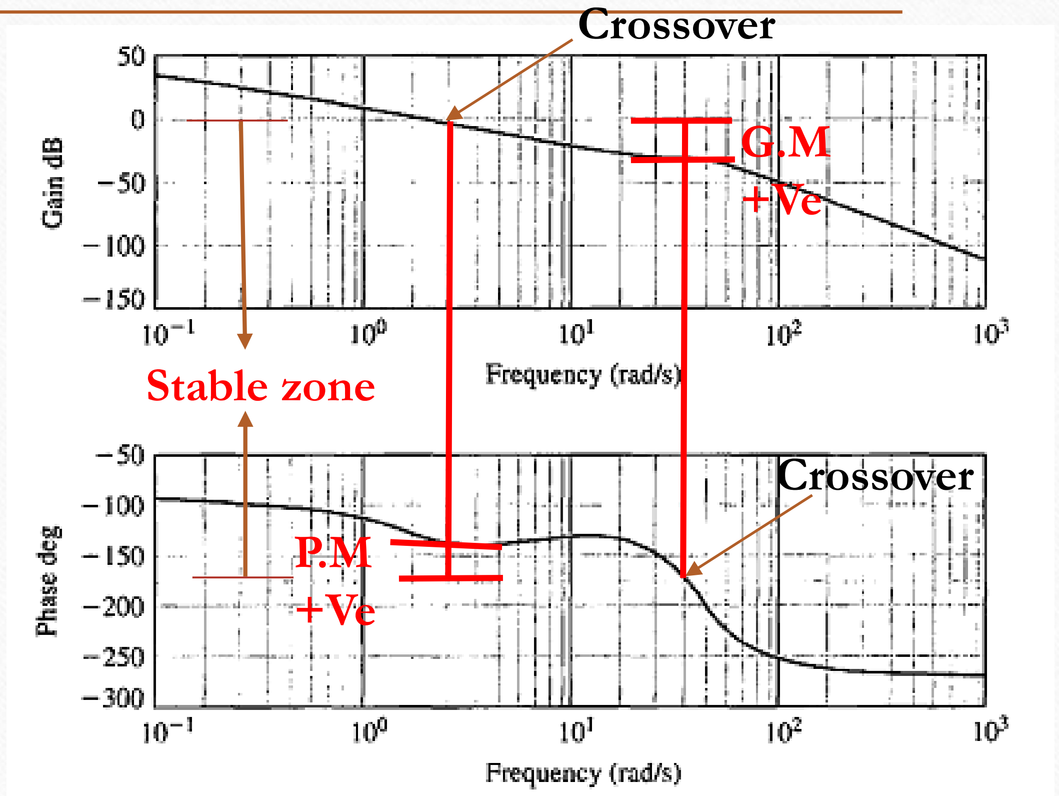
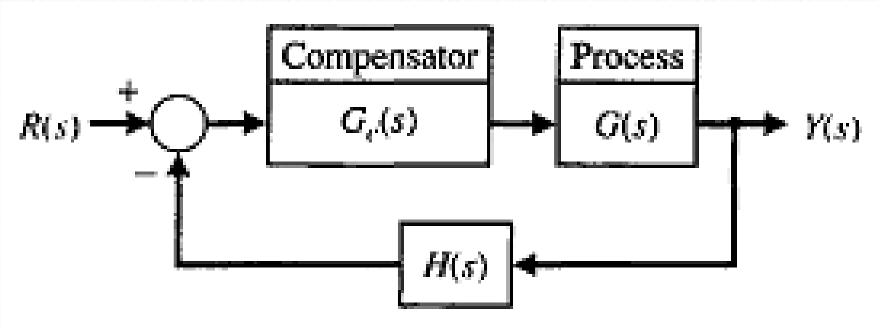


Design of Compensator

- **A compensator is an additional component or circuit that is inserted into a control system to compensate for a deficient performance**

The transfer function of a compensator is designated as $G_c(s) = E_0(s)/Ei(s)$, and the compensator can be placed in a suitable location within the structure of the system

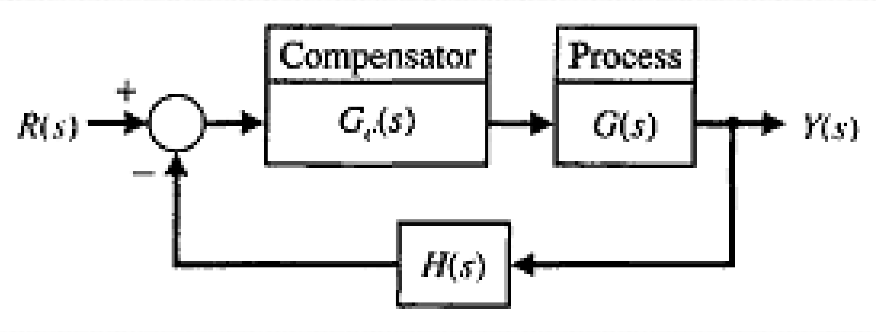


Phase-lead network

- The design problem then becomes the selection of z ,p, and K in order to provide a suitable performance.
- The **first-order compensator** with the transfer function

$$G_c(s) = \frac{K(s+z)}{s+p}.$$

A compensator *Gc(s)* is used with a process *G(s)* so that the overall loop gain can be set to satisfy the steady-state error requirement, and then *Gc(s)* is used to adjust the system dynamics



Phase-lead network

lead compensator for a system

- Steady-state error for a ramp input $K_v = \frac{A}{e_{ss}}$

The velocity constant of a type-one uncompensated $K_v = \lim_{s \to 0} s\{G(s)\},$

$$G(s) = \frac{K \prod_{i=1}^{M} (s + z_i)}{s \prod_{j=1}^{n} (s + p_j)},$$

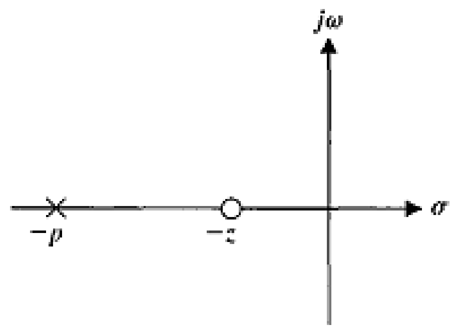
$$K_v = \frac{K \prod_{i=1}^{M} z_i}{\prod_{j=1}^{n} p_j}.$$

Phase-lead network

- If $|p| >> |z|$, and the zero occurred at the origin of the s-plane

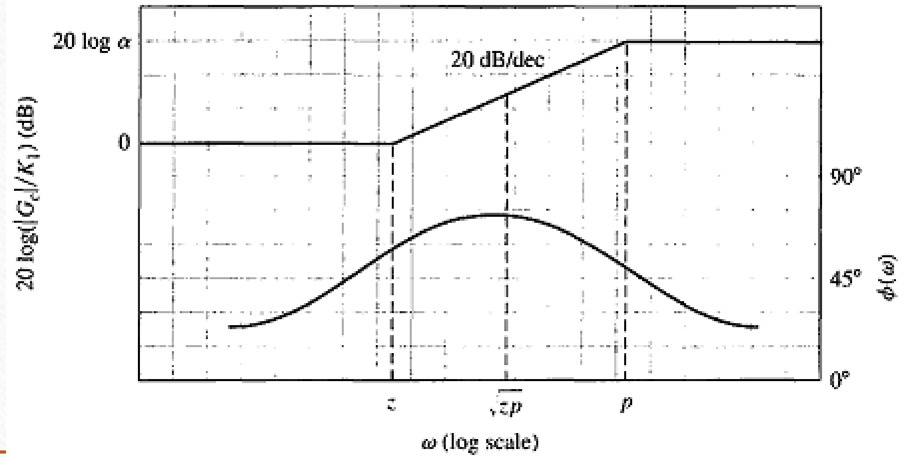
$$G_c(s) \approx \frac{K}{p} s.$$

- we would have a differentiator, and the frequency characteristic and a phase angle of $+90°$



$$G_c(j\omega) = \frac{K(j\omega + z)}{j\omega + p} = \frac{(Kz/p)[j(\omega/z) + 1]}{j(\omega/p) + 1} = \frac{K_1(1 + j\omega\alpha\tau)}{1 + j\omega\tau},$$

where $t = 1/p$, $p = \alpha z$, and $K1 = K/\alpha$.



Phase-lead network

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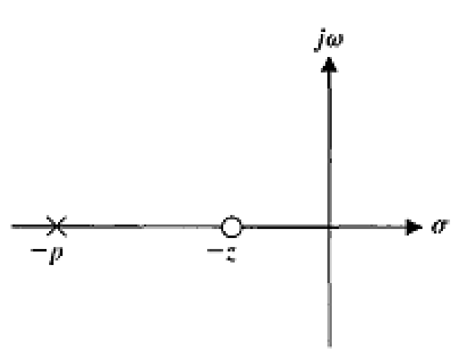


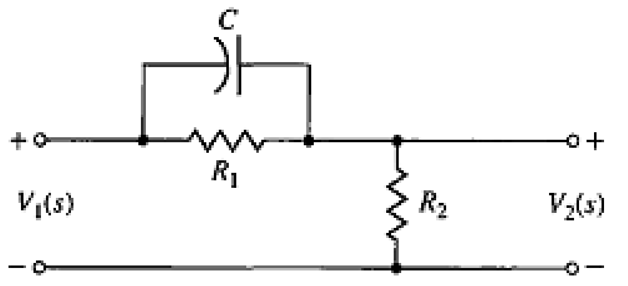
$$G_c(s) \approx \frac{K}{p} s.$$

The angle of the frequency characteristic is

$$\phi(\omega) = \tan^{-1}(\alpha\omega\tau) - \tan^{-1}(\omega\tau)$$

Because the zero occurs first on the frequency axis, we obtain a phase-lead characteristic. The slope of the asymptotic magnitude curve is +20 dB/decade

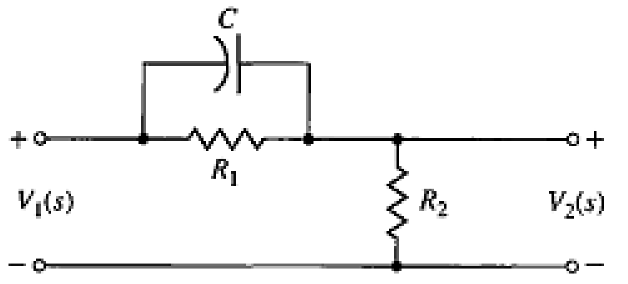
The phase-lead compensation transfer function can be obtained with the network shown in Figure



Phase-lead network

- If $|p|>>|z|$, and the zero occurred at the origin of the s-plane

$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \dfrac{R_1/(Cs)}{R_1 + 1/(Cs)}} = \frac{R_2}{R_1 + R_2}\frac{R_1Cs + 1}{[R_1R_2/(R_1 + R_2)]Cs + 1}.$$



we let

$$\tau = \frac{R_1R_2}{R_1 + R_2}C \quad \text{and} \quad \alpha = \frac{R_1 + R_2}{R_2},$$

we obtain the **phase-lead compensation** transfer function

$$G_c(s) = \frac{1 + \alpha\tau s}{\alpha(1 + \tau s)},$$

The maximum value of the phase lead occurs at a frequency ω_m,

The maximum phase lead occurs halfway between the pole and zero frequencies

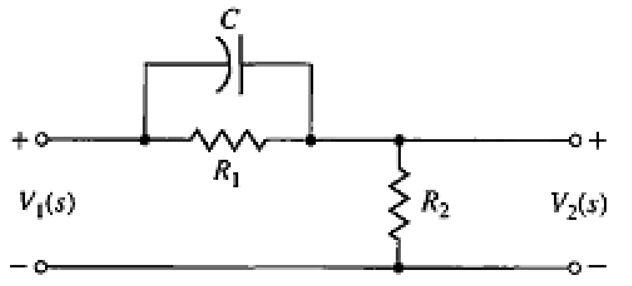
$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$

Phase-lead network

- If $|p| >> |z|$, and the zero occurred at the origin of the s-plane

To obtain an equation for the maximum phase-lead angle

$$\phi = \tan^{-1}\frac{\alpha\omega\tau - \omega\tau}{1 + (\omega\tau)^2\alpha}.$$

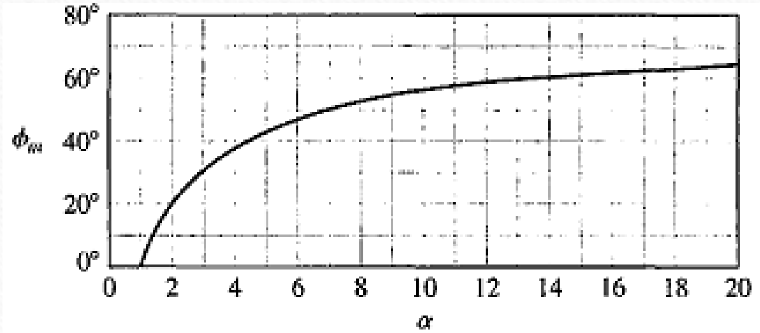


substituting the frequency for the maximum phase angle, ω_m

$$\tan\phi_m = \frac{\alpha/\sqrt{\alpha} - 1/\sqrt{\alpha}}{1 + 1} = \frac{\alpha - 1}{2\sqrt{\alpha}}.$$

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}.$$

There are practical limitations on the maximum value of α that we should attempt to obtain. if we required a maximum angle greater than 70°, two cascade compensation networks would be used.



Phase-lead network

Determine the compensation network by completing the following steps:

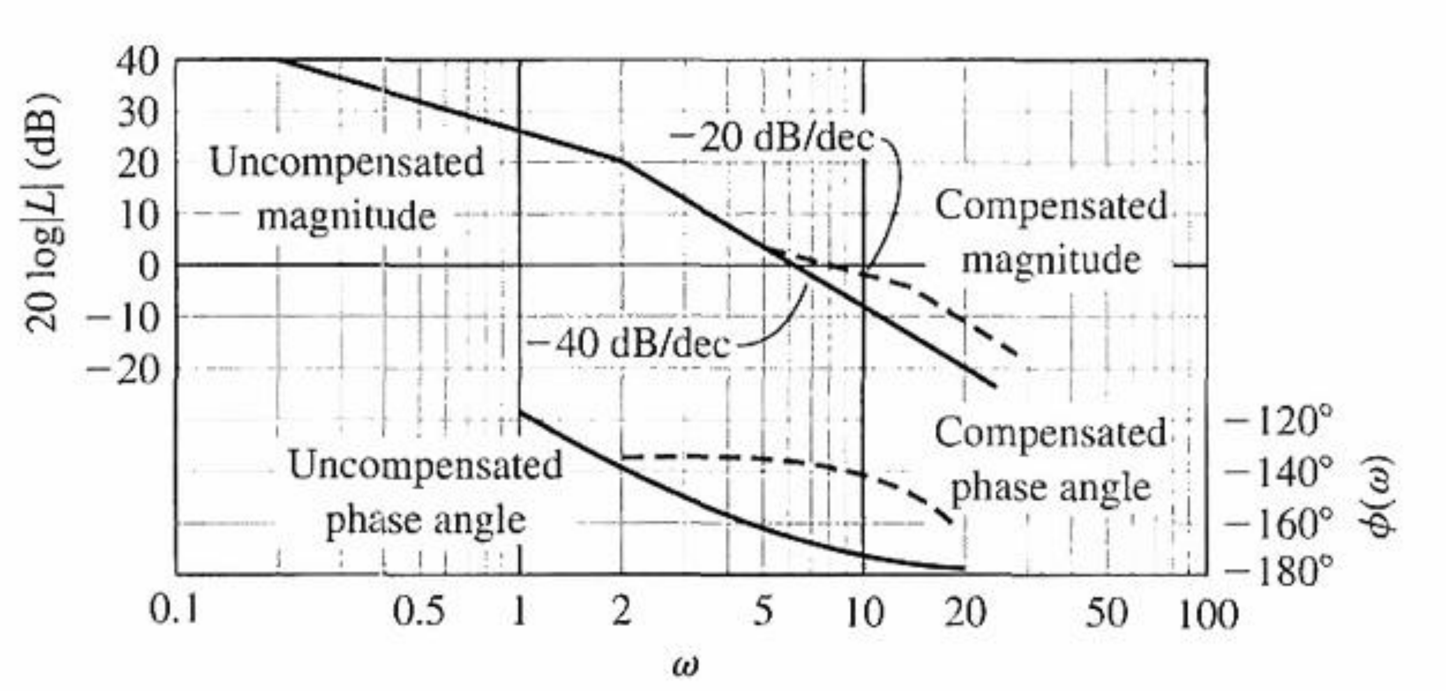
1. Evaluate the **uncompensated** system phase margin when the error constants are satisfied.
2. Allowing for a small amount of safety, **determine** the necessary additional **phase** lead φ_m.
3. **Evaluate α** from Equation.

$$\sin \phi_m = \frac{\alpha - 1}{\alpha + 1}.$$

4. Evaluate **10 log α** and determine the frequency where the uncompensated magnitude curve is equal to -10 log α dB. Because the compensation network provides a gain of 10 log α at ω_m, this frequency is the new 0-dB crossover frequency and ω_m simultaneously.
5. **Calculate** the pole $p = \omega_m \sqrt{\alpha}$ and $z = p/\alpha$.
6. **Draw** the **compensated** frequency response, **check** the resulting phase margin, and repeat the steps if necessary. Finally, for an acceptable design, raise the gain of the amplifier in order to account for the attenuation (1/ α).

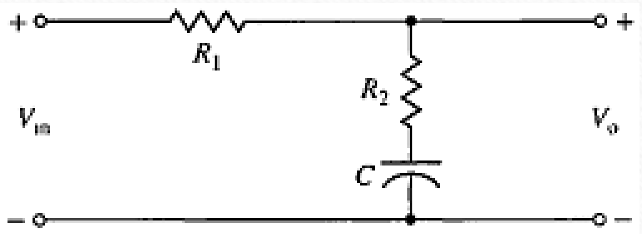
Phase-lead network

Determine the compensation network by completing the following steps:



Phase-lag network

- The transfer function of the **phase-lag** network is ***RC* network** suitable for compensating a feedback control



$$G_c(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_2 + 1/(Cs)}{R_1 + R_2 + 1/(Cs)} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}.$$

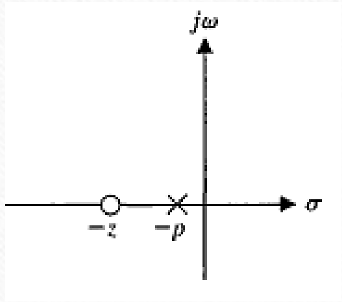
When $t = R_2C$ and $\alpha = (R_1 + R_2)/R_2$,

where $z = 1/t$, $p = 1/(\alpha t)$.

$$G_c(j\omega) = \frac{1 + j\omega\tau}{1 + j\omega\alpha\tau}$$

We will now add the **integration-type** phase-lag network as a compensator and determine the compensated velocity constant.

$$G_c(s) = \frac{1 + \tau s}{1 + \alpha\tau s} = \frac{1}{\alpha}\frac{s + z}{s + p},$$



The maximum phase lag occurs at ω_m

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$

Phase-lag network

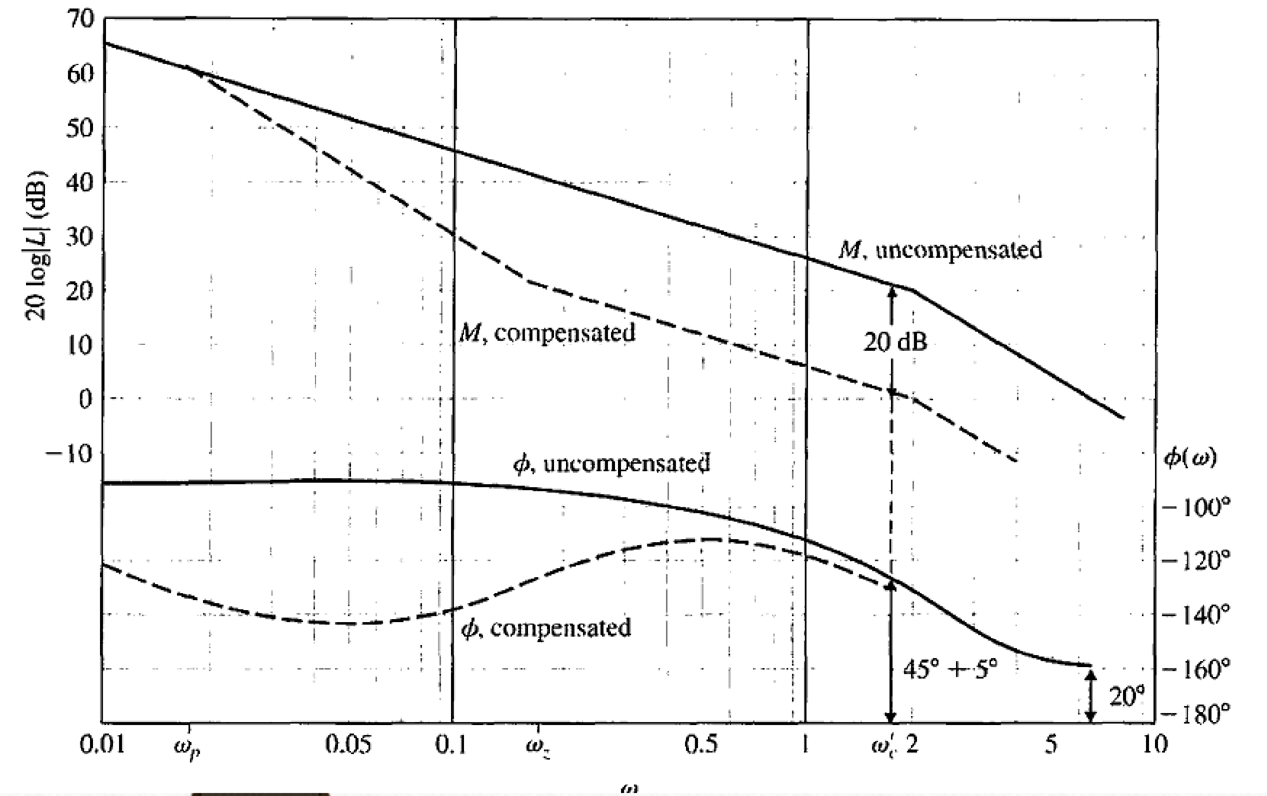
Determine the compensation network by completing the following steps:

1. **Obtain** the Bode diagram of the **uncompensated** system with the gain adjusted for the desired error constant.
2. **Determine** the phase margin of the **uncompensated** system and, if it is insufficient, proceed with the following steps.
3. **Determine** the frequency where the **phase margin** requirement would be satisfied if the magnitude curve crossed the **0-dB** line at this frequency, $\omega'c$. (Allow for 5° phase lag from the phase-lag network when determining the new crossover frequency.)
4. **Place** the zero of the compensator one decade below the new crossover frequency $\omega'c$, and thus ensure only 5° of additional phase lag at $\omega'c$

5. **Measure** the necessary attenuation at $\omega'c$ to ensure that the **magnitude** curve crosses at this **frequency**.
6. **Calculate** α by noting that the attenuation introduced by the phase-lag network is —20 log α at $\omega'c$.
7. **Calculate** the **pole** as $\omega p = 1/(\alpha t) = \omega_z / \alpha$, and the design is completed

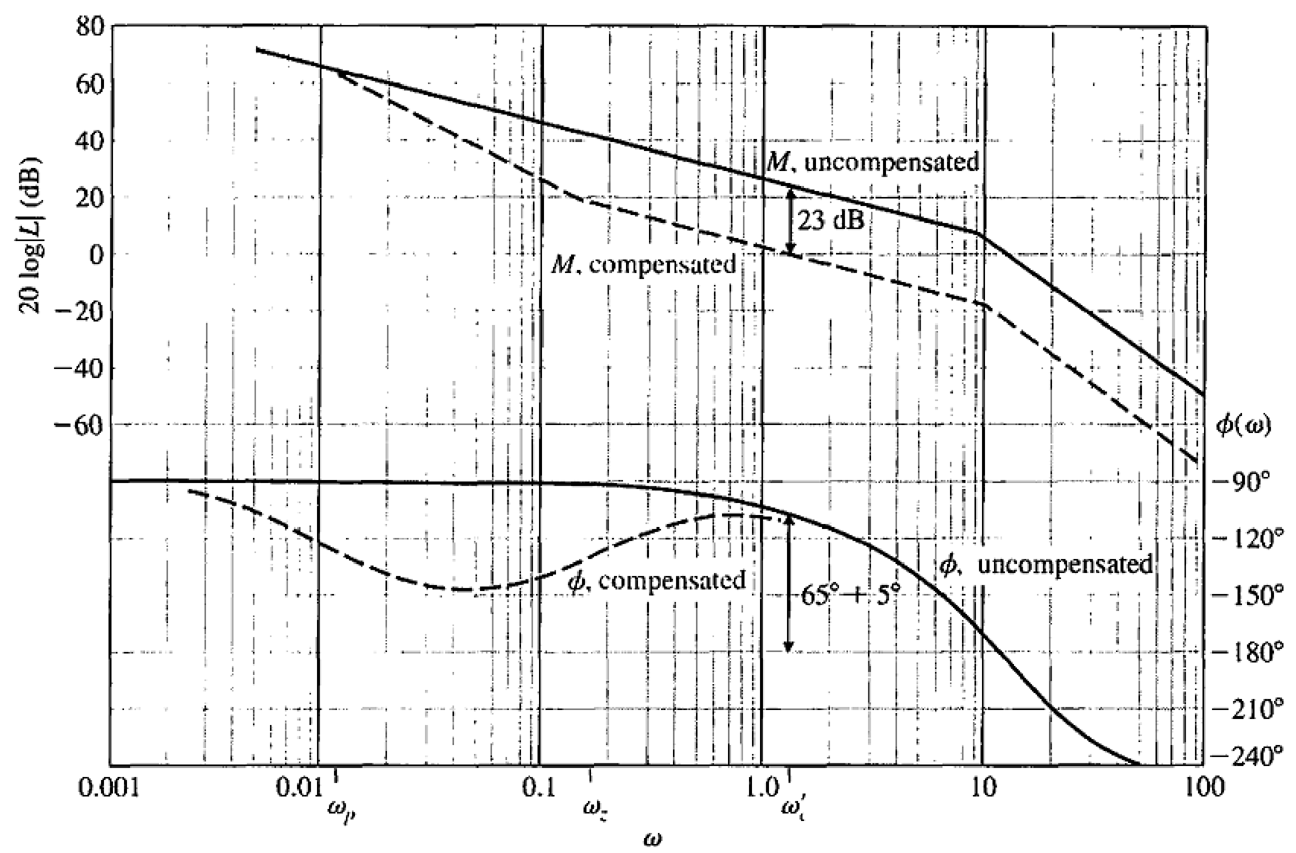
Phase-lag network

Determine the compensation network by completing the following steps:



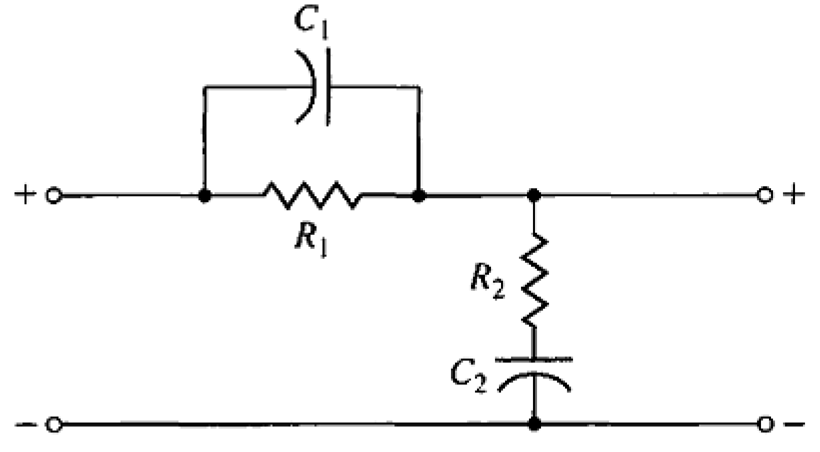
Phase-lag network

Determine the compensation network by completing the following steps:



Lead-lag network

The phase-lead compensation network alters the frequency response of a network by adding a positive (leading) phase angle and therefore increases the phase margin at the crossover (0-dB) frequency. An analytical technique of selecting the parameters of a lead or lag network has been developed for the Bode diagram



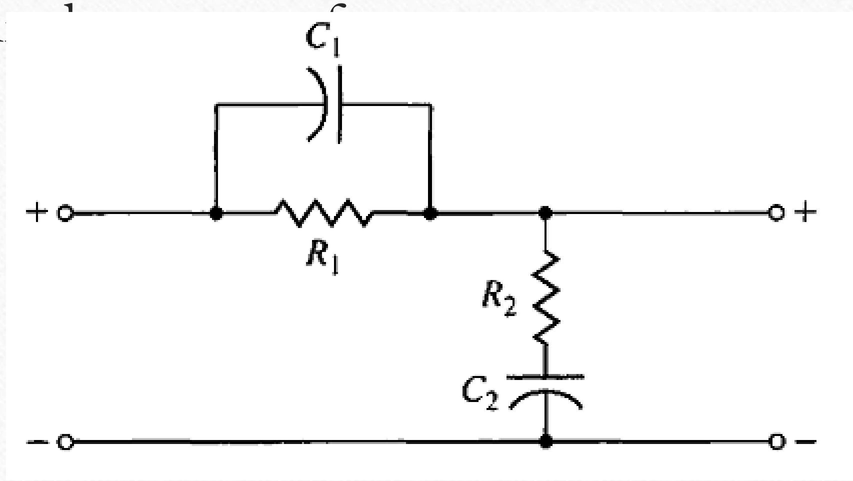
$$\frac{V_2(s)}{V_1(s)} = \frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2)s + 1}.$$

$$\frac{V_2(s)}{V_1(s)} = \frac{(1 + \alpha\tau_1 s)(1 + \beta\tau_2 s)}{(1 + \tau_1 s)(1 + \tau_2 s)}$$

Lead-lag network

- If α < 1 yields a **lag** compensator and α > 1 yields a **lead** compensator.
- The **phase contribution** of the compensator at the desi

$$p = \tan\phi = \frac{\alpha\omega_c\tau - \omega_c\tau}{1 + (\omega_c\tau)^2\alpha}.$$



- The **magnitude *M*** (in dB) of the compensator at

$$c = 10^{M/10} = \frac{1 + (\omega_c\alpha\tau)^2}{1 + (\omega_c\tau)^2}.$$

- obtain the nontrivial solution equation for α **as**

$$(p^2 - c + 1)\alpha^2 + 2p^2c\alpha + p^2c^2 + c^2 - c = 0.$$

- single-stage **compensator**, it is necessary that *c* > *p2* + 1

$$\tau = \frac{1}{\omega_c}\sqrt{\frac{1 - c}{c - \alpha^2}}.$$

Lead-lag network

Determine the compensation network by completing the following steps:

1. Select the desired ω_c.
2. Determine the **phase** margin desired and therefore the required phase φ .

$$p = \tan\phi = \frac{\alpha\omega_c\tau - \omega_c\tau}{1 + (\omega_c\tau)^2\alpha}.$$

3. Verify that the **phase lead** is applicable: φ > 0 and $M > 0$.
4. Determine whether a single stage will be sufficient by testing $c > p2 + 1$.
5. Determine **α** from. $(p^2 - c + 1)\alpha^2 + 2p^2c\alpha + p^2c^2 + c^2 - c = 0.$
6. Determine **t** from. $\tau = \frac{1}{\omega_c}\sqrt{\frac{1 - c}{c - \alpha^2}}.$

If we need to design a single-lag compensator, then φ < 0 and M < 0 (step 3).
Step 4 will require $c < 1/(1 + p^2)$.